IB MYP YEAR 5

ASSESSMENT TASK

A Broad-based Test

Subject:	Y10 ***Extended*** Mathematics	**Name : (Class)**	Jessi Lui (10J)
Assessment:	**Broad-based Test**		
Topics covered	**Polynomials, Transformations (and transforming functions), Probability, Vectors, Matrices, Indices**		
Date of assessment:	**Thursday 16th February 2012**		

- **This task assesses Criteria A and C;**
- **Time allowed – *one hour 40 minutes;***
- **You must answer <u>all</u> the questions;**
- **Write your answers in the spaces provided;**
- **Show all of your working – <u>not</u> just the answer**
- **GDCs are allowed.**

Criterion A

Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	Students are reasonably successful with the Part A questions only. Any errors here are relatively minor.	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.
3-4	Students are successful with Part A questions. The only errors in part B questions are minor.	The student generally makes appropriate deductions when solving **more complex** problems in **familiar** contexts.
5-6	Students are successful with Part A and B questions. The only errors in part C questions are minor.	The student **generally** makes appropriate deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7-8	Students are successful with Part A, B and C questions. The only errors in part D questions are minor.	The student **consistently** makes appropriate deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar situations.**

Criterion C

Levels	Task-Specific Rubric	Official IB Descriptors
0	The student does not reach a standard described by any of the descriptors given below.	
1-2	Very little working is shown, and/or the steps shown are confusing. Only the most basic mathematical symbols are used with accuracy.	The student shows **basic use** of mathematical language and/or forms of mathematical representation. The lines of reasoning are **difficult to follow.**
3-4	The working shown is generally adequate. Only a few errors in symbols/terminology are evident. It is reasonably easy to follow a student's logic/reasoning.	The student shows **sufficient use** of mathematical language and forms of mathematical representation. The lines of reasoning are **clear though not always logical or complete**. The student moves between different forms of representation with **some success.**
5-6	There are very few, if any, errors in symbols/terminology. All steps in calculations are shown in their completeness. It is easy to follow all the student's logic/reasoning.	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise, logical** and **complete.** The student moves **effectively** between different forms of representation.

Part A (Level 1-2 Questions)

Q1. Factorise ***completely*** the following expressions:

(a) $x^2 - 16$

$(x+4)$
$(x-4)$

$x = 4$ or $x = -4$

Answer (a) 4 or -4

(b) $x^2 - 16x$

$x^2 - 2 \cdot x \cdot 8 + 8^2 = 0 + 8^2$

$(x+8)^2 = 64$

$x+8 = \pm\sqrt{64}$ $x = 8 \pm \sqrt{64}$

$x = 16$

Answer (b) 16 or 0

(c) $x^2 - 16x - 36$

$$x = \frac{16 \pm \sqrt{256 - 4(1)(-36)}}{2(1)}$$

$x = 18$ or -2

$$x = \frac{16 \pm \sqrt{400}}{2} \qquad x = \frac{16 \pm 20}{2}$$

Answer (c) 18 or -2

Q2. Simplify the following expressions, giving your answers in the form a^n, where a and n are integers:

(a) $3^5 \times 3^{-4}$

$3^{5+(-4)} = 3^1$

Answer (a) $3^1 = 3$

(b) $2^4 \times 3^4$

$= 6^4$

Answer (b) 6^4

(c) $(6^2)^3$

$= 6^{2\times3} = 6^6$

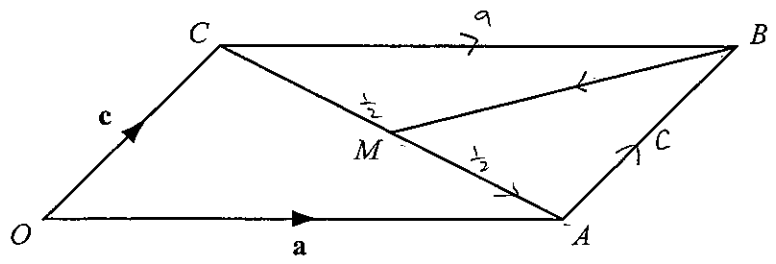
Answer (c) 6^6

(d) $16^2 \div 2^8$

$$\frac{16^2}{2^8} = \frac{2^{4(2)}}{2^8} = \frac{2^8}{2^8} = 1$$

Answer (d) 2^0

Q3.



OABC is a parallelogram. $\overrightarrow{OA}$ = **a**, $\overrightarrow{OC}$ = **c** and M is the midpoint of CA. Find, in terms of **a** and **c**:

(a) $\overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

Answer (a) $c + a$

(b) $\overrightarrow{CA}$

$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$

Answer (b) $-c + a$

(c) $\overrightarrow{BM}$

$\overrightarrow{BM} = \overrightarrow{MA} + \overrightarrow{AB}$

$= \overrightarrow{MA} + c$

$= \frac{1}{2}\overrightarrow{CA} + c$

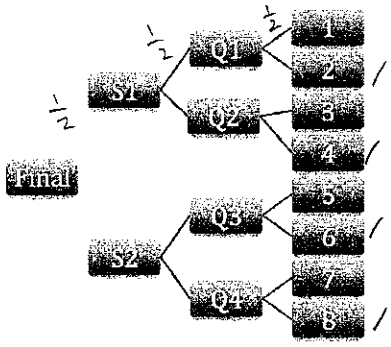
$= \frac{1}{2}(-c + a) + c$

$= -\frac{1}{2}c + \frac{1}{2}a + c$

Answer (c) $\frac{1}{2}c + \frac{1}{2}a = \frac{1}{2}(a + c)$

Part B (Level 3-4 Questions)

Q4. Eight teams take part in a basketball tournament. Each team is equally likely to win any particular game. Winning teams advance to the next stage. The losers go home!!



(a) Find the probability that all the even numbered team reach the quarter-finals (Q1, Q2, Q3, Q4).

Probability for team 2, 4, 6, & 8 to reach quarter-finals

$P = \left(\frac{1}{2} \cap \frac{1}{2} \cap \frac{1}{2} \cap \frac{1}{2}\right)$

$= \frac{1}{16}$

Answer (a) $\frac{1}{16}$

(b) Find the probability that team 1 will play team 8 in the final.

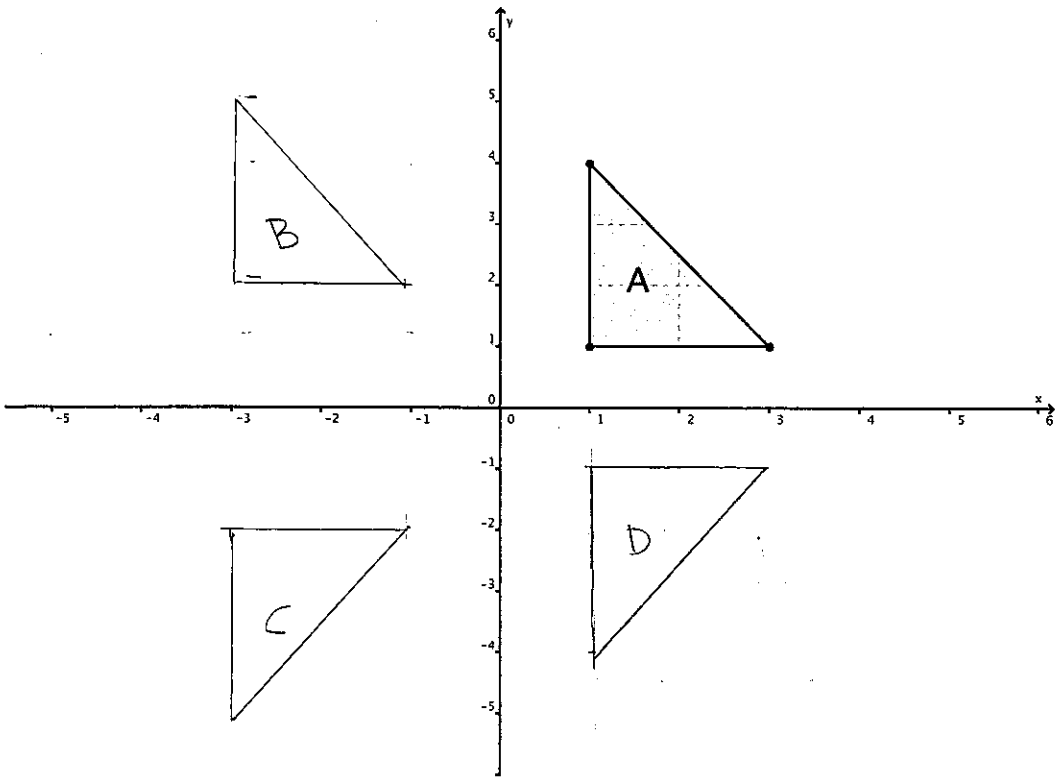
$P(\text{Team 1 play in final} \cap \text{Team 8 play in final})$

$= \frac{1}{8} \times \frac{1}{8}$

$= \frac{1}{64}$

Answer (b) $\frac{1}{64}$

Q5. Triangle A is plotted on the Cartesian plane below.



(a) On the same Cartesian plane above, draw the following:

(i) Triangle A after it is translated by a translation vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$, label it B.

(ii) Triangle C is the reflection of triangle B in the *x*-axis, draw triangle C.

(iii) Triangle D is the reflection of triangle A in the *x*-axis, draw triangle D.

(b) Describe the single transformation, which maps triangle C onto triangle D.

Answer $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ .....

Q6. Erik runs a race at an average speed of x m/s.
His time is $(3x - 9)$ seconds and the race distance is $(2x^2 - 8)$ metres.

(a) Write down an equation in x and show that it simplifies to
$x^2 - 9x + 8 = 0$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$x = \frac{2x^2 - 8}{3x - 9}$$

$$3x - 9(x) = 2x^2 - 8$$

$$3x^2 - 9x = 2x^2 - 8$$

$$x^2 - 9x + 8 = 0$$

(b) Solve $x^2 - 9x + 8 = 0$

$$x = \frac{9 \pm \sqrt{81 - 4(1)(8)}}{2}$$

$$= \frac{9 \pm \sqrt{49}}{2}$$

$$= \frac{9 \pm 7}{2}$$

$x = 8$ or $x = 1$

Answer (b) 8 or 1

(c) Write down Erik's time and the race distance.

Time = $(3x - 9)$ s

$= [3(8) - 9]$ s or $[3(1) - 9]$ s

$= 15$ s $= -6$ s (Not possible)

Distance:

$(2x^2 - 8)$ m $(2x^2 - 8)$ m

$= 2(8)^2 - 16$ or $= 2(1)^2 - 8$

$= 2(64) - 16$ $= 2 - 8$

$= 128 - 16$ $= -6$ m (Not possible)

$= 112$ m

Answer (c) 15 seconds and 112 meters

Part C (Level 5-6 Questions)

Q7. Luis deposits a large sum of money in a bank account that pays 0.6% interest, compounded monthly. How long does it take Luis's money to grow by 10%?

$$A = P\left(1 + \frac{0.006}{12}\right)^{12t}$$

$$1.1x = x\left(1 + \frac{0.006}{12}\right)^{12t}$$

$$1.1x = x(1.0005)^{12t}$$

$$1.1 = (1.0005)^{12t}$$

$$12t = 200$$

$$t = 16.6$$

$$A = P(1+r)^t$$

$$1.1x = x(1 + 0.006)^t$$

$$1.1x = x(1.0006)^t$$

$$t = 16$$

Answer ..... 16 years .....

Q8. A is the matrix $\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix}$ and AB is the matrix $\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$. Find the matrix B.
(Remember the order of matrix multiplication matters!)

$$\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5a+2c & 5b+2d \\ 2a+0c & 2b+0d \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5a+2c & 5b+2d \\ 2a & 2b \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5(22)+2c & 5(4)+2d \\ 2(22) & 2(4) \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$$

$a = 22$
$b = 4$
$c = -\frac{99}{2}$
$d = -9$

Answer ..... $\begin{pmatrix} 22 & 4 \\ -\frac{99}{2} & -9 \end{pmatrix}$ .....

Q9. Solve the following equation for x:

$4^{x+1} + 4^x + 4^{x-1} = 42$

$$4^x\left(4 + 1 + 4^{-1}\right) = 42$$

$$4^x\left(\frac{21}{4}\right) = 42$$

$$4^x = 8$$

$$2^{2x} = 8$$

$$2^{2x} = 2^3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Answer .....

Part D (Level 7-8 Questions)

Q10. You are given the quadratic equation $x^2 + kx + 4 = 0$

(a) What values of k ensure that this equation has repeated real roots?

$x = \frac{-k \pm \sqrt{k^2 - 16}}{2}$

$2x = -k \pm \sqrt{k^2 - 16}$

$\Delta = b^2 - 4ac = 0$ for repeated real roots

$k^2 - 4(4) = 0$

$k^2 = 16$

$k = \pm\sqrt{16}$

Answer (a) ..... 4 or −4

(b) It is decided that k is an integer. A student chooses a value of k at random from the interval

$-5 \leq k \leq 5$

What is the probability that the resulting quadratic equation has no real roots?

$\Delta = b^2 - 4ac$

$\Delta = b^2 - 4ac < 0$ (NRS)

$\Delta = b^2 - 4ac \geq 0$ (YES)

$-5^2 - 16 = -41$
$-4^2 - 16 = 0$ ✱
$-3^2 - 16 = -7$
$-2^2 - 16 = 12$
$-1^2 - 16 = -17$
$0^2 - 16 = -16$
$1^2 - 16 = -15$
$2^2 - 16 = -12$
$3^2 - 16 = -7$
$4^2 - 16 = 0$ ✱
$5^2 - 16 = 9$ ✱

Probability (no real roots) = 1 − Probability (yes real roots) ✱

$= 1 - \frac{3}{11}$

$= \frac{8}{11}$

Answer (b) ..... $\frac{8}{11}$

(c) You are told to transform the original quadratic to $y = x^2$ **in one step.** What value of k would you choose, and what transformation would you perform?

Choosing $k = 4$:

$y = x^2 + 4x + 4$

$= (x+4)^2$

Transformation:

Horizontal Translation to the right by 4 units

Choosing $k = -4$.

$y = x^2 - 4x + 4$

$= (x-4)^2$

Transformation:

Horizontal Translation to the left by 4 units

Answer (c) ..... $k = 4$, move curve to right by 4 units so that original quadratic will transform to $y = x^2$

Q11. $A = \begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} x \\ y \end{pmatrix}$

We are told that AB = kB (where k is an integer).

(a) Using AB = kB, set up and simplify two simultaneous equations with x and y in.

$$\begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = k\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+6y \\ 4x+3y \end{pmatrix}$$

$$M^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

$$= \frac{1}{3-(24)}\begin{pmatrix} 3 & -6 \\ -4 & 1 \end{pmatrix}$$

$$= \frac{1}{-21}\begin{pmatrix} 3 & -6 \\ -4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x+6y \\ 4x+3y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

$$\frac{1}{-21}\begin{pmatrix} 3 & -6 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = k\begin{pmatrix} x \\ y \end{pmatrix}\frac{1}{-21}\begin{pmatrix} 3 & -6 \\ -4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = k\begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} -\frac{1}{7} & \frac{2}{7} \\ \frac{4}{21} & -\frac{1}{21} \end{pmatrix}$$

$x + 6y = kx$

$4x + 3y = ky$ ★ Answer

$x - kx = -6y$

$4x = ky - 3y$

(b) By solving the simultaneous equations in (a), find possible values for k.

$1 + 6 = 7$

$1.5 + 6 = k(1.5)$

$= 5$

$x + 6y = kx \quad 4 + 3 = ky$

$4x + 3y = ky \quad k = 7$

$7 + 3 = ky$

$2x^2 + xy - 3y = 0$

$x = 1$ or

$x = 1.5$

$y = 1$

-3

$\frac{x+6y}{x} = \frac{4x+3y}{y}$

$yx + 6y^2 = 4x^2 + 3xy$

$6y^2 - 4x^2 = 2xy$

$4x^2 + 2xy - 6y^2 = 0$

$x(1-k) = -6y$

$x = \frac{ky-3y}{4}$

$\frac{-6}{1-k} = \frac{y(k-3)}{4}$

$-24 = 1-k(yk-3y)$

$-24 = yk(1-k) - 3y(1-k)$

$-24 \quad yk - yk^2 - 3y + 3yk$

$-24 = -y^2 + 4yk - 3y$

$-24 = y(-x^2 + 4k - 3)$

$(2x + 3y)(2x - 3y)$

$x = \frac{3y}{2}$

$x = \frac{-2y}{2}$

$\frac{3y}{2} + 6y = k\left(\frac{3y}{2}\right)$

$2x + 12x = 6ky$

$x + 6 = \frac{-24}{-x^2+4k-3}$

$x^2 + 6x = \frac{144}{6x - 24k + 18}$

$-x^2 + 4k - 3 = y$

$4k(3x + 12x) = 6ky$

$12ky + 18ky = 6kxy$

Answer ..... 1 or 7.9 .....

NOW GO BACK AND CHECK YOUR WORK